Cherry Maps with Different Critical Exponents: Bifurcation of Geometry

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Abstract We consider order preserving C^3 circle maps with a flat piece, irrational rotation number and critical exponents (ℓ_1, ℓ_2).

We detect a change in the geometry of the system. For $(\ell_1, \ell_2) \in [1, 2]^2$ the geometry is degenerate and it becomes bounded for $(\ell_1, \ell_2) \in [2, \infty)^2 \setminus \{(2, 2)\}$. When the rotation number is of the form $[abab \cdots]$; for some $a, b \in \mathbb{N}^*$, the geometry is bounded for (ℓ_1, ℓ_2) belonging above a curve defined on $]1, +\infty[^2$. As a consequence we estimate the Hausdorff dimension of the non-wandering set $K_f = \mathcal{S}^1 \setminus \bigcup_{i=0}^{\infty} f^{-i}(U)$. Precisely, the Hausdorff dimension of this set is equal to zero when the geometry is degenerate and it is strictly positive when the geometry is bounded.

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