STATISTICAL STRUCTURES ARISING IN NULL SUBMANIFOLDS

ABSTRACT. We show a link between affine differential geometry and null submanifolds in a semi- Riemannian manifold via statistical structures. Once a rigging for a null submanifold is fixed, we can construct a semi-Riemannian metric on it. This metric and the induced connection constitute a statistical structure on the null submanifold in some cases. We study the statistical structures arising in this way. We also construct statistical structures on a null hypersurface in the LorentzMinkowski space using the null second fundamental form. This extends the classical construction to the null case.

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