# $C^0$-transport of flux geometry

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ABSTRACT

The goal of this paper is to study, in a large scale point of view, the flux geometry of a closed symplectic manifold $(M, \omega)$: namely, the topological counterpart of the flux homomorphism. Using metrics arising from the decomposition of closed 1-forms with respect to an arbitrary linear section $\mathcal{S}$, we generalize the construction of the group of strong symplectic homeomorphisms. The flux homomorphism for symplectomorphisms is extended to a surjective group homomorphism $S^0_\omega$ on the group of $\mathcal{S}$-homeomorphisms. We prove that the kernel of $S^0_\omega$ is path connected, coincides with the subgroup $Hameo(M, \omega)$ of all Hamiltonian homeomorphisms and investigate the discreteness of the corresponding flux group $\mathcal{S}\Gamma_\omega$. Later on, without appealing to any lifting map, we give an alternative proof of a result from the classical flux geometry saying that any smooth symplectic isotopy in $Ham(M, \omega)$ is a Hamiltonian isotopy. Furthermore under some hypothesis, we prove that any $\mathcal{S}$-topological isotopy in $Hameo(M, \omega)$ is a continuous Hamiltonian isotopy. We also proved that any $\mathcal{S}$-topological isotopy with trivial flux is homotopic to a continuous Hamiltonian isotopy, relatively to fixed endpoints.