The Unicity of a Blow-up.

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Abstract.

Typical of the classification problem in Mathematics is the classification of non-singular varieties within a given birational equivalence class in **Birational Geometry**. Towards solving the problem is the resolution of singularities. One of the techniques used in the process is **blowing-up**. Blowing up a variety at a point or a subvariety gives a non-singular or a less singular variety. In this research, we investigate if the blow-up is unique. Precisely, let X be a variety with a non-singular curve $C \subset X$ and $f : X \longrightarrow S$ be a birational morphism mapping X to another variety S. Suppose that the map f **contracts** the curve C to another curve $D \subset S$ of a lower dimension. And, given a blow up $\pi : \widetilde{S} \longrightarrow S$, does there exist a morphism $g : X \longrightarrow \widetilde{S}$ leading to an isomorphism?. In other words, is the map f also a blowing up?